B.A/B.Sc. 5th Semester (Honours) Examination, 2023 (CBCS)

Subject : Mathematics

Course : BMH5DSE23

(Boolean Algebra and Automata Theory)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Notation and symbols have their usual meaning.

1. Answer *any ten* questions: 2×10=20

(a) Let L be a lattice and $a, b, \in L$. Then show that $a \leq b$ implies $a \vee c \leq b \vee c$.

(b) Determine the minterm normal form of the Boolean expression $f(x_1, x_2) = x_1 \vee x_2$.

(c) Define a recursive language and give an example.

(d) Construct a finite automata equivalent to the regular expression: $L = a\,a^*\,(a + b)^*$.

(e) Give an example of a lattice where distributive laws do not hold.

(f) State the recognition problem.

(g) Construct a logic circuit that produces $(x + y + z)(xyz)'$ as its output.

(h) What is pushdown automata? Give an example.

(i) Give an example of Turing Machine that accepts the 'Empty Language'.

(j) Draw a transition diagram for a machine that recognizes whether or not a string in B^* contains an even number of 1's.

(k) What is NFA?

(l) What is ambiguous grammar?

(m) Define down-sets and give an example.

(n) Define a string and give an example.

(o) Find the simplified form of the Boolean function: $a + a\bar{b}$.

2. Answer *any four* questions: 5×4=20

(a) Show that every regular language is a content-free language.

(b) Show that the set of Turing-machine codes for Turing Machines that accept all inputs that are palindromes (possibly along with some other inputs) is undecidable.

(c) Construct a PDA to accept the following language: $L = \{a^n b^{2n} | n \geq 1\}$.

(d) Transform the following DNF to CNF:

$$x_1' x_2' x_3' + x_1' x_2' x_3 + x_1 x_2' x_3' + x_1 x_2 x_3'$$

(e) Design a Turing Machine that computes the function $f(x,y) = x+y$ if $x \geq y$
$= 0$ if $x < y$.

(f) Show that in a complemented distributive lattice the following are equivalent:

(i) $a \leq b$

(ii) $a \wedge b' = 0$

(iii) $a' \vee b = 1$

(iv) $b' \leq a'$

3. Answer *any two* questions: 10×2=20

(a) (i) Design a Turing machine that accepts the language $L = \{0^{2^n} | n \geq 0\}$.

(ii) Prove that for any transition function δ and for any two input strings x and y $\delta(q, xy) = \delta(\delta(q,x), y)$. 6+4

(b) (i) Suppose a 3-variable Boolean term is given as follows:

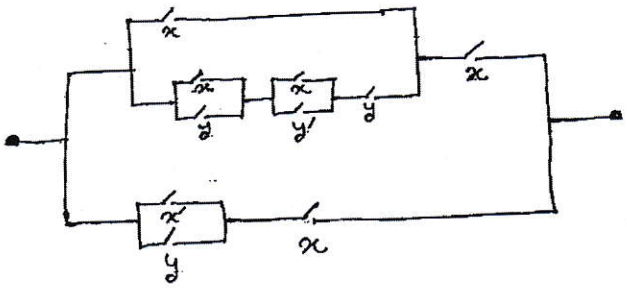
$\emptyset = xy + xz' + yz$.

Minimize ϕ using K-map.

(ii) Suppose $\emptyset(a,b,c,d) = \Sigma m(0,1,3,7,8,9,11,15)$. Minimize ϕ using Quine-McCluskey method. 5+5

(c) (i) Prove that a lattice L is distributive if and only if $x \wedge (y \vee z) \leq (x \wedge y) \vee z$, for all $x, y, z, \in L$.

(ii) Obtain a Boolean expression which represents the following circuit. Moreover, draw an equivalent circuit as simple as you can: 5+5



(d) Discuss about the equivalences of deterministic and non-deterministic automata and find the equivalent deterministic finite automata for the given non-deterministic finite automata: 5+5